1. Swell/Consolidation

The software's calculations are based for the most part on ASTM D2435 with some modifications to make certain data optional, and to add partial support for ASTM D4546 (specifically, methods A and B). As D2435 concerns itself with hand-plotted – as opposed to mathematically-modeled – curves, the program also makes choices concerning curve modeling that are outside of scope of the specification.

The following sections detail the calculation procedures utilized by the software. Note that the equations assume that dimensions are given in centimeters and weights are in grams.

1.1 Calculation of Initial Specimen Parameters

Moisture, density, initial void ratio and saturation values are calculated from the measurements on the sample. There are some variations among laboratories in the procedures followed when recording these measurements. Briefly, the different procedures handled by the program and what effect they have on calculations are as follows:

1. The dimensions and weight of a larger sample, not the actual test specimen, are recorded. The height of the actual test specimen is also recorded and entered in the program. In this case, the weight of the test specimen is found by multiplying the weight of the larger sample by the ratio of test specimen volume to the volume of the larger sample.
2. The actual test specimen dimensions and weight are recorded and entered in the program. This is the simplest case and the program calculates the moisture, density, saturation, etc. directly from the measurements.
3. As an alternative to using the weight of solids calculated by the software from the initial specimen, a dried post-test specimen weight can be entered.

Initial Moisture Content

The initial (natural) moisture content is calculated with the following formula:

$$MC_i = \frac{W_{wt} - W_{dt}}{W_{dt} - W_t} * 100 \tag{1.1}$$

Where:

MC_i = the moisture content
W_{wt} = the weight of the moisture specimen with tare
W_{dt} = the weight of the dried specimen with tare
W_t = the weight of the container

Wet Density

The initial wet (natural) and dry densities are calculated from the untrimmed specimen as follows:

$$\gamma_{wi} = \frac{W_t}{V_t} \tag{1.2}$$

Where:

γ_{wi} = the initial wet (natural moisture) density
W_t = the specimen weight
V_t = specimen volume

$$V_t = \frac{\pi * D_t^2 * H_t}{4} \tag{1.3}$$

Where:

D_t = the specimen diameter
H_t = the specimen height

Dry Density

$$\gamma_{di} = \frac{\gamma_{wi}}{1 + \frac{MC_i}{100}} * \left(\frac{D_c}{D_t}\right)^2 * \frac{H_c}{H_t} \tag{1.4}$$

Where:

γ_{di} = the sample dry density
D_c = the consolidometer ring diameter
H_c = the height of the sample in the consolidometer

Initial Void Ratio

The initial void ratio is calculated from the test specimen size and mass as follows:

$$e_0 = \frac{H_t - H_s}{H_s} \tag{1.5}$$

Where:

e_0 = the initial void ratio
H_t = the specimen test height
H_s = the height of solids

$$H_s = \frac{W_d}{G_s} * \frac{1}{A_t} \tag{1.6}$$

and

Where:

W_d = the weight of mineral solids in the test specimen (calculated from either the pre-test or post-test sample)

G_s = the specimen specific gravity

A_t = the area of the tested specimen

Mineral Solids Weight

W_d, the weight of the mineral solids, may be determined either from the pre-test or post-test sample, depending upon whether the **Use final solids weight instead of estimated initial weight** box has been checked:

Use final solids weight... unchecked:

$$W_d = \frac{W_t}{1 + \frac{MC_i}{100}} \tag{1.7}$$

Use final solids weight... checked:

$$W_d = W_{fdt} - W_{ft} \tag{1.8}$$

Where:

W_{fdt} = the dried weight of the post-test specimen

W_{ft} = the post-test specimen container weight

Initial Saturation

$$S_i = \frac{MC_i * W_d}{(H_t - H_s) * A_t} * 100 \tag{1.9}$$

Where:

S_i = the initial saturation in percent

1.2 Post-Test Calculations

A post-test moisture-content test is not mandatory; however, if performed, *and the entire test specimen is used for the moisture content test*, the final dried sample weight may be used as W_d.

Final Moisture Content

$$MC_f = \frac{W_{fwt} - W_{fdt}}{W_{fdt} - W_{ft}} * 100 \tag{1.10}$$

Where:

MC_f = the final specimen moisture content
W_{fwt} = the final weight of the wet specimen with tare
W_{fdt} = the final weight of the dried specimen with tare
W_{ft} = the weight of the container

Final Saturation

$$S_f = \frac{MC_f * W_d}{H_f - H_s} * 100 \tag{1.11}$$

Where:

$$H_f = H_i - \delta H \tag{1.12}$$

δH = the total change in height during the test

1.3 Intermediate (Per-Loading Increment) Calculations

Void ratios and percent strain are calculated at the end of each loading increment using the following equations:

End-of-Loading Increment Void Ratio

The void ratio at the end of a loading increment is:

$$e_n = e_0 - \delta e \tag{1.13}$$

Where:

e_n = the void ratio at the end of loading increment number *n*
δe = the change in void ratio from the initial sample to the end of loading increment *n*

and

$$\delta e = \frac{DR_n - DR_0}{H_s} \tag{1.14}$$

Where:

DR_n = the loading increment's corrected final dial reading

DR_0 = the dial reading at test start (time $t = 0$ for the first loading increment)

DR_n is calculated differently depending upon both the test data and the **Use D_{100} values** selection:

If **Use D_{100} values** is *not* checked:

$$DR_n = DR_x - MD_n \tag{1.15}$$

Where:

DR_x = the final dial reading recorded during the loading increment

MD_n = the machine deflection selected for the loading increment, or 0 if either no machine deflection table was selected for the test, or no deflection was specified in the deflection table for loading increment *n*.

If **Use D_{100} values** is checked *and a D_{100} result could be calculated* for the loading increment:

$$DR_n = D_{100n} \tag{1.16}$$

Where:

D_{100n} = the loading increment's D_{100} result, as calculated via either the $\log_{10}$(time) or square-root(time) construction methods

If **Use D_{100} values** is checked *and a D_{100} result could not be calculated* for the loading increment (either because time-rate data were not entered for the increment, or the time-rate constructions could not be placed in a way that would produce a D_{100} value, or the **Report** button was unchecked on both the $\log_{10}$(time) or square-root(time) charts for the increment):

$$DR_n = DR_x - (DR_{xp} - D_{100p}) \tag{1.17}$$

Where:

DR_{xp} = the final dial reading of the first *prior* loading increment for which a D_{100} value could be calculated

D_{100p} = the calculated D_{100} result for the same loading increment

A compression curve plotted using D_{100} values omits the effects of secondary (i.e., post-D_{100}) consolidation, while one plotted using final dial readings (DR_n) incorporates the effects of both primary and secondary consolidation. Finally, a compression curve plotted using a mix of D_{100} and DR_n values conflates the two results – to combat this, the program attempts to synthesize a D_{100} value for non-D_{100} loading increments by assuming a similar amount of secondary consolidation for a non-D_{100} loading increment as occurred in the last loading increment for which a D_{100} value could be calculated (secondary consolidation being the $(DR_{xp} - D_{100p})$ term in the previous equation).

⇒ This is undoubtedly at best a poor approximation. It is suggested that the **Use D_{100} values** option *not* be selected unless D_{100} results can be obtained for all loading increments.

End-of-Loading Increment Percent Strain

$$S_n = \frac{DR_n - DR_0}{H_0} \tag{1.18}$$

Where:

S_n = the strain at the end of loading increment *n*, in percent
H_0 = the initial height of the test specimen

Percent Heave

If the **ASTM D4546 compatibility** option is selected the program will calculate a percent heave value for each loading increment.

$$HV_n = \frac{DR_n - DR_i}{H_t - DR_i} \tag{1.19}$$

Where:

HV_n = the percent heave at the end of loading increment *n*
H_i = the height of the sample at the time saturation
DR_n = the sample deformation at the end of the loading increment
DR_i = the sample deformation at the time of saturation

The "modified" version of ASTM D4546 Method A consists of the following sequence of steps:

1. A preliminary loading increment at seating pressure
2. Load to overburden pressure
3. Unload to seating pressure
4. An inundation loading increment

If this sequence is detected, the heave equation is:

$$HV_n = \frac{DR_n - DR_o}{H_t - DR_o} \tag{1.20}$$

Where:

DR_o = the sample deformation at the end of loading increment 2 (the "load to overburden pressure" increment)

1.4 Compression Curve Calculations

Compression Curve Plotting Model

Values of void ratio, strain percentage or deformation are plotted against $\log_{10}(pressure)$ using a specialized curve modeling method that eliminates a problem associated with traditional cubic spline compression curve models, namely that due to model's smoothness requirements, cubic-spline modeled compression curves will show swell (or consolidation) between two adjacent points at similar deformations (such as typically occurs between the chart left side and the first, seating, load when using the "plot starting at load = 0" option). In other words, a curve segment plotted between two loading increments with minimal deformation between the increments should be linear, but will typically exhibit phantom swell or compression between the points when modeled using the traditional cubic spline approach.

Compression Index

The compression index (C_c) is defined as the slope of the straight-line portion of the compression curve:

$$C_c = \frac{\delta e}{\log_{10}(\frac{\sigma_2}{\sigma_1})} \tag{1.21}$$

Where:

C_c = the compression index (always a positive value)
δe = the void ratio change between σ_1 and σ_2
σ_1 = the pressure at one end of the straight line section
σ_2 = the pressure at the other end

Preconsolidation Pressure

CONS calculates preconsolidation pressures (P_c) using the standard Casagrande method, outlined below:

1. The point of maximum curvature on the curve is located
2. A tangent to the curve at that point is determined
3. The horizontal line passing through that same point is drawn
4. The equation for the line bisecting the above lines is determined
5. The point is located where the tangent to the linear portion of the virgin consolidation curve meets this bisector line (the same tangent is used that was selected for the C_c): P_c is equal to the pressure corresponding to this point.

Recompression/Swell Indices

CONS calculates and reports a value for either C_r, the recompression index, or C_s, the swell index, depending upon whether or not a saturation cycle was included as part of the test: C_s is calculated for tests including a saturation cycle, C_r is calculated otherwise.

- C_s is the average of the slopes of the unloading cycle segments of the compression curve
- The C_r calculation deptends upon the **Recompression index is calculated as** configuration option: C_r will be either the average slope of the reload portions of the curve, or the average of the unload and reload slopes.

If an overburden pressure is entered *and the preconsolidation pressure is greater than the overburden pressure* the slope averaging will only be performed on the portion of the curve between the overburden and preconsolidation pressure.

Wetting Cycles

CONS can proceses data from tests that involve sample saturation, including calculating the following values:

$$\epsilon_s = \epsilon_2 - \epsilon_1 \tag{1.22}$$

Where:
ϵ_s = the swell strain, in percent (collapse gives negative results)
ϵ_1 = the strain percentage at the end of the load prior to wetting
ϵ_2 = the strain percentage at the end of the innudation load

Swell pressure is calculated variously based upon the **Swell pressure is calculated as** configuration setting (either as the pressure to return the sample to the pre-wetting height, or that pressure *minus* the pre-wetting loading pressure.

⇒ In either case the software will not be able to report a swell pressure unless the sample is re-compressed to at least the pre-wetting height. Samples that collapse during wetting will naturally not have a swell pressure reported.

1.5 Log(time) Per-Loading Increment Calculations

$\log_{10}(time)$ curves are modeled using a fourth-order polynomial regression curve.

- The program uses the root of the derivitive of the resulting curve equation to determine the point of maximum slope. The initial left-hand construction line placement is tangent to this point.
- The right-most construction line is initially placed by building a linear regression fit through all of the points past the curve's final bend (which represents the transition between primary and secondary consolidation). The initial fit is iteratively reweighted to eliminate outliers.

D_0 is calculated variously based upon the **D_0 is** selection:

- If the selection is **Construction**, D_0 is the deformation at the intersection of the left-hand **t-4t** construction, minus the change in deformation between the **4t** curve intersection and the **t** curve intersection.
- D_0 is the deformation at the first reading entered for the load if **Initial dial** is selected.

D_{100} is the intersection of left and right-hand construction lines.

$$D_{50} = \frac{D_0 + D_{100}}{2} \tag{1.23}$$

T_{50} is the time value of the intersection of a horizontal line drawn through D_{50} with the $\log_{10}(time)$ polynomial equation.

C_v, the coefficient of consolidation, is:

$$c_v = \frac{T * H_p^2}{T_{50}} \tag{1.24}$$

Where:

C_v = the coefficient of consolidation for the loading increment
T = a dimensionless time factor = 0.197
H_p = the average drainage path length during the loading increment

and:

$$H_p = \frac{H_0 - \delta H_t - \frac{\delta H_n}{2}}{DF} \tag{1.25}$$

Where:

H_0 = the initial specimen height
δH_t = the total specimen deformation up to the start of loading increment n
DF = the drainage path factor = $\begin{cases} 1 & \text{single drainage tests} \\ 2 & \text{double drainage tests} \end{cases}$
δH_n = the change in specimen height during loading increment n
$= \begin{cases} D_{100} - D_0 & \text{if } \textbf{Use D100 values} \text{ is checked} \\ DR_{nf} - DR_{n0} & \text{if } \textbf{Use D100 values} \text{ is unchecked} \end{cases}$
DR_{n0} = the initial dial reading from loading increment n
DR_{nf} = the final dial reading for loading increment n

C_α, the coefficient of secondary consolidation, is calculated variously based upon the **Secondary compression is calculated as** configuration option as the slope of the right-hand C_v construction divided by either H_s (the height of solids) or H_0 (the initial specimen height).

1.6 Square-Root(time) Per-Loading Increment Calculations

$\sqrt{time}$ curves are plotted using either a point-to-point method or a sixth order polynomial regression fit.

⇒ $\ln_2$(time) vs. $\ln_2$(dial reading) is used when creating the regression fit as the transformation creates an approximately linear curve, which can make for a better approximation.

- The left-hand tangent line, which should be placed tangent to the points representing the early time readings that exhibit a straight line trend, is placed by iteratively attempting to model the entire curve with three regression lines (one lying tangent to the initial straight line portion, the second tangent to the "bend" in the curve at the end of the initial straight line portion, and the third tangent to the final, mostly secondary consolidation section of the curve). When the best placement has been located (as determined by the placement that results in the smallest sums of the squares of the residuals of the fit), the left-most of the tangent lines is used as the left-hand construction line.
- The right-hand tangent line intersects the left-hand construction line at time = 0 with a slope that is 1.15 times that of the left.
- D_0 is the dial reading at which the left and right construction lines intersect (i.e., at time = 0).
- The T_{90} vertical line is placed at the intersection point of the right-hand construction line with the curve. D_{90} is the dial reading at this intersection point.

$$D_{100} = D_0 - \frac{10}{9} * (D_0 - D_{90}) \tag{1.26}$$

Where:

D_{100} = the dial reading at 100 % primary consolidation

$$C_v = \frac{TF_x H_p^2}{T_x} \tag{1.27}$$

Where:

C_v = the coefficient of consolidation
H_p = the average drainage path, as defined in the previous section